Assessment of cast-iron columns using analytical models

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\textbf{Abstract.} The paper focuses on the assessment of cast-iron columns in industrial heritage structures. For cast-iron structures it is difficult to verify metallurgical composition and processing technology, which directly affect the geometry of cross-sections. The crucial issue of reliability of cast-iron structures is their brittle fracture in tension at higher slenderness ratios. The load carrying capacity of columns is affected by their stability and cast-iron strength in compression and tension. Outcomes of a recently proposed analytical model are compared with extensive experimental data and model uncertainty is quantified. It appears that the model is in a good agreement with experimental data and provides reasonably conservative estimates of load carrying capacity. Outcomes of the study may provide basis for foreseen improvements in calculating buckling coefficient according to ČSN 730038:2014.

1. \textbf{Introduction}

In the 19th and early 20th century cast iron was popular construction material for columns and beams (compressed part), utilising its favourable mechanical properties. In present projects of rehabilitation of industrial heritage structures such as textile factories, railway stations or bridges, assessment of the load carrying capacity and remaining working life of cast-iron columns is a crucial issue. It is widely recognised that these columns hardly fulfil requirements of present standards. A key step of reliability assessment is modelling of resistance of load-bearing members made of cast iron. The present contribution investigates two analytical models for resistance of historic cast-iron columns. Outcomes of the models are critically compared with experimental results obtained for solid and hollow cylindrical, and square columns from English grey cast iron (see Section 3). Imprecision of the models is expressed by means of model uncertainty for which appropriate probabilistic models are proposed.

2. \textbf{Model uncertainty}

The concept of the model uncertainty proposed in [1, 2, 3] is adopted here. The uncertainties in resistance models are obtained from comparisons of physical tests and model results; real structure-specific conditions need then to be taken into account when they significantly deviate from test conditions. General framework of the uncertainty assessment for models of cast-iron columns with examples of influences affecting test and model results is given in Figure 1. Computational options seem to be irrelevant in this study since simple analytical models are considered.

Treatment of the test uncertainty was proposed in [2]. It was shown that unbiased test results with coefficient of variation around 0.05 can be assumed for tests of common reinforced concrete members. In the absence of statistical data these indications are accepted for cast-iron columns. The test uncertainty was proved to be of low significance and negligible when higher coefficient of variation of model uncertainty (say, greater than 0.1) is observed [2]. As this is the case in the present study, the test uncertainty is hereafter neglected.
The model uncertainty $\theta$ is here treated as a random variable. The multiplicative relationship for $\theta$ is assumed [4]:

$$R(X,Y) = \theta(X,Y) R_{\text{model}}(X)$$ (1)

where $R$ = response of a structure - real resistance estimated from test results; $R_{\text{model}}$ = model resistance - estimate of the resistance based on a model; $X = \text{vector of basic (random) variables included in the model}$; and $Y = \text{vector of variables neglected in the model, but possibly affecting the resistance}$. Modulus of elasticity is the example of a variable $Y$ for some models for resistances of cast-iron columns.

3. Properties of cast iron

Cast iron is generally an alloy of iron, carbon and other elements. Carbon is present in the form of graphite and its content should exceed 2.1%. Cast irons are classified according to the shape of expelled graphite:

- Ductile cast iron (spheroidal graphite)
- Grey cast iron (lamellar graphite)
- White cast iron (graphite has been excluded and remain bound in iron)
- Malleable cast iron (flake graphite).

Industrial heritage structures were mostly made of grey cast iron, characterized by a high carbon content from 3.5% to 5% and by a minimum amount of other constituents. Relatively high content of graphite aggregated in lamellas leads to stress concentrations, decreases plasticity and thereby significantly reduces tensile strength that is up to four times lower than compressive strength of cast iron.

Design procedures provided in EN 1993-1-1: 2006 "Design of Steel Structures" [5] cannot be directly applied to evaluate cast-iron structures due to significantly different stress-strain diagrams of steel and cast-iron, Fig. 2.
It can be observed from Fig. 2 that the stress-strain diagram of cast iron is:

- Similar to those of aluminum and stainless steels rather than to mild steel,
- Non-linear, resulting in reduced buckling strength when the material gradually loses its stiffness,
- Without unambiguous yield stress, thus stress $f_{0.2}$ corresponding to plastic (permanent) deformation of 0.2% is often introduced,
- Indicating much lower tensile strength than compressive strength due to higher carbon content leading to brittleness of the material.

4. **Specifics cast-iron as a construction material**

Load capacity centrically loaded columns is highly dependent on the slenderness of columns and geometric parameters of their cross-sections. The imperfections are mostly caused by unknown technology of casting such as hand casting or forging. Due to casting in a horizontal position cross sections have inner eccentricities and different wall thicknesses. Together with lack of straightness these imperfections govern the stability of slender columns.

Model proposed in [6] determines strength $\sigma_{\text{model}}$ of cast-iron columns exposed to buckling as a minimal value of its compressive $\sigma_c$ and tensile strength $\sigma_t$:

$$\sigma_{\text{model}} = \min(\sigma_c; \sigma_t)$$ (2)

Tensile strength becomes decisive for columns with a high slenderness ratio.

Two models, denoted hereafter as **Approach 1** and **Approach 2**, can be used to estimate compressive strength. Using **Approach 1** [6], $\sigma_c$ is obtained as:

$$\sigma_c = \chi_c \times \sigma_{0.2}$$ (3)

where $\sigma_{0.2}$ = nominal strength based on the stress-strain curve proposed in [7]; and $\chi_c$ = slenderness reduction factor obtained similarly as recommended in EN 1993-1-1:2006 [5] with considerations for specific properties of cast iron. The nominal strength of 375 MPa is recommended for cast iron [6]. For low slenderness ratios, $\lambda < 25$, **Approach 1** numerically fails as the reduction factor exceeds unity. In such cases $\sigma_c = \sigma_{0.2}$ is here taken into account. However, these cases are of low practical significance.

**Approach 2** [8] is valid for any slenderness ratio:

$$\sigma_c = \frac{552 \text{ MPa}}{1 + \lambda^2 / 1600}$$ (4)
The tensile strength is assessed as follows [6]:

\[ \sigma_t = \chi_t \times f \times \sigma_{0.2} \]  \hspace{1cm} (5)

where \( \chi_t \) = reduction factor accounting for slenderness ratio; and \( f \) = ratio between tensile and compressive strength. Equation (5) apparently takes basis in \( \text{Approach 1} \). The representative value \( f = 0.2 \) is accepted in [6] as a conservative value for English grey iron. In practical cases it is recommended to derive a value of the parameter \( f \) from tensile tests.

Assuming \( \sigma_{0.2} = 375 \text{ MPa} \), \( f = 0.2 \) and the reduction factors \( \chi_c \) and \( \chi_t \) according to [6], it can be shown that:

- For \( \lambda \leq 37 \), \( \sigma_c \) obtained by \( \text{Approach 1} \) is negligibly lower (by about 2%) than that based on \( \text{Approach 2} \),
- For \( 37 < \lambda < 66.5 \) \( \text{Approach 2} \) leads to \( \sigma_c \)-values lower than \( \text{Approach 1} \); the maximum difference of 10% is observed for \( \lambda \approx 50 \); the difference vanishes with increasing slenderness ratio,
- A limiting value of slenderness ratio above which tensile strength becomes decisive for \( \sigma_{\text{model}} \) in Equation (2) is \( \lambda_{\text{lim}} = 55.7 \) for \( \text{Approach 1} \) and \( \lambda_{\text{min}} = 66.5 \) for \( \text{Approach 2} \).

5. Database of experimental results

Uncertainty assessment for the considered models is based on comparison of test and model outcomes. Database of experimental results includes 72 tests of cast-iron columns with different slenderness ratios. The outcome of a test \( \sigma_{\text{test}} \) represents compressive stress corresponding to a force causing the failure of a specimen. All columns have been made of English grey iron with the expected content of carbon between 3.5-5% and small amount of additives. The content of carbon is dependent on a manufacturing process. The database is divided into three samples according to cross sections of the columns (Tab. 1).

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Sample size</th>
<th>Slenderness ratio</th>
<th>Column strength [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid cylindrical</td>
<td>50</td>
<td>26-242</td>
<td>14.8-537</td>
</tr>
<tr>
<td>Hollow cylindrical</td>
<td>18</td>
<td>50.8-242</td>
<td>31.9-186</td>
</tr>
<tr>
<td>Solid square</td>
<td>4</td>
<td>154-204</td>
<td>24.2-43.6</td>
</tr>
</tbody>
</table>

The database includes solid and hollow cylindrical columns with slenderness ratios uniformly covering the range from 26 to 242 (Fig. 3, tab. 1). The sample for solid square columns is small \( n = 4 \); only specimens with high slenderness ratio are included. The database contains no information about cross-section characteristics, eccentricities and imperfections.

Fig. 3: Variation of \( \sigma_{\text{test}} \) with \( \lambda \) for solid cylindrical columns
6. Statistical evaluation of model uncertainty

Values of model uncertainties are obtained from equation (1). In tab. 2, the statistical parameters for individual cross-sections are divided by slenderness ratio and models. Tensile strength is dominant for higher slenderness ratio than $\lambda \geq 50.8$.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Model</th>
<th>Sample size</th>
<th>$\lambda$</th>
<th>Mean $\mu_\theta$</th>
<th>Coefficient of variation $V_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid cylindrical</td>
<td>$\text{Approach 1} (\sigma_c)$</td>
<td>7</td>
<td>26.5 – 55.7</td>
<td>1.18</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\sigma_t^*$</td>
<td>43</td>
<td>55.7 – 242.4</td>
<td>1.2</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>$\text{Approach 2} (\sigma_c)$</td>
<td>12</td>
<td>26 – 66.5</td>
<td>1.11</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\sigma_t^*$</td>
<td>38</td>
<td>66.5 – 242.4</td>
<td>1.24</td>
<td>0.11</td>
</tr>
<tr>
<td>Hollow cylindrical</td>
<td>$\sigma_t$</td>
<td>18</td>
<td>50.8 – 242.4</td>
<td>1.12</td>
<td>0.11</td>
</tr>
<tr>
<td>Solid square</td>
<td>$\sigma_t$</td>
<td>4</td>
<td>153.8 – 204.1</td>
<td>1.43</td>
<td>0.08</td>
</tr>
</tbody>
</table>

It follows from Table 2 that the model for tensile strength given in Equation (5) is more conservative ($\mu_\theta \approx 1.12-1.43$) than $\text{Approaches 1 and 2}$ for compressive strength ($\mu_\theta \approx 1.11-1.18$). This indicates that the considered value $f = 0.2$ be inappropriate for the investigated database and should be revised. Dispersion of model uncertainty as expressed by its coefficient of variation ranging mostly between 0.1 and 0.15 corresponds well to buckling resistance of steel columns [9, 10]. However, the sample sizes for $\text{Approaches 1 and 2}$ and solid cylindrical columns and for tensile strength and solid square columns are small and obtained characteristics of model uncertainty should be considered as indicative only.

Taking into account the limited amount of data, the following recommendations are provided on the basis of the results given in Table 2:

- Model uncertainty characteristics $\mu_\theta \approx 1.2$ and $V_\theta \approx 0.15$ should be considered when compressive strength is decisive in Equation (2),
- $\mu_\theta \approx 1.25$ and $V_\theta \approx 0.15$ should be considered when tensile strength is governing resistance of a cast-iron column.

These characteristics can be directly applied when deriving model uncertainty factor for assessments using the partial factor method as provided in $EN \ 1990:2002$ for basis of structural design [11, 12].

Figure 4 shows variation of model uncertainty values with slenderness ratio for solid cylindrical columns. $\text{Approaches 1 and 2}$ seem to be conservative particularly for low slenderness ratios, $\lambda < 30$. However, these cases are rare in practical situations. In most cases $\lambda > 70$ applies and the model for tensile strength is decisive for resistance of columns. Figure 4 indicates that this model may be also conservative with considerable dispersion of outcomes. The conservative bias may be reduced by specifying an appropriate value of the ratio $f$. The dispersion is attributed to varying effects of eccentricities and imperfections that seem to be inadequately taken into account by the model for tensile strength. A more advanced model is proposed in [13]. The considered models may overestimate real resistances for $55.7 < \lambda < 66.5$ when compressive and tensile strengths become comparable.
7. Conclusions

Reliability verification of existing cast-iron columns is often a key issue in rehabilitations of industrial heritage structures. Uncertainties in resistance models can then become a crucial aspect of such verifications. Design procedures provided in EN 1993-1-1: 2006 cannot be directly applied to evaluate cast-iron structures due to significantly different stress-strain diagrams of steel and cast-iron.

As the tensile strength of cast iron is considerably lower than compressive strength, it is a variable dominating resistances of centrically loaded columns with slenderness ratio over 60. In such cases model uncertainty can be described by a two-parameter lognormal distribution [4] with the mean of 1.25 and coefficient of variation of 0.15. For columns with lower slenderness ratios compressive strength is decisive and the mean of model uncertainty decreases to 1.2.

In further research obtained findings will be utilised in improvements of the procedure for verification of cast-iron columns according to ČSN 730038:2014.

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References


