Probabilistic Assessment of Historic Reinforced Concrete Bridge

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Keywords: Bridge, reinforced concrete and probabilistic assessment.

Abstract. This paper is aimed at the reliability analysis of an existing reinforced concrete bridge from 1908. The load bearing capacity is assessed in accordance with valid standards using the partial factor method and probabilistic approach. Load bearing capacities obtained by these methods are critically compared. The application of probabilistic method leads to 40 % higher load bearing capacity then the partial factor method used for structural design.

Introduction

More than 50 % of investments in construction are related to existing structures. This ratio is even greater in bridge engineering due to continuous degradation, ever increasing traffic intensities and general lack of financial resources for rehabilitations of bridges. That is why effective assessment of the load bearing capacity of existing bridges is becoming a crucial issue. In regard to this the present study is aimed at the probabilistic assessment of historic reinforced concrete bridge and at the comparison with results obtained by the partial factor method.

Final report COST Action [1] estimates that more than million bridges exist in the 27 European countries and it represents approximately 400 billion Euros of replacement costs. Therefore, even small improvements in the methodology of assessment could lead to substantial savings. The qualified decisions about replacement or upgrade of bridges should be based on the available information and actual state of the bridge, unfavourable effects of environment and potential consequences due to malfunction of the bridge.

The case study is focused on the bridge built in 1908. The bridge is chosen on the basis of complexity of available information about geometry and material properties. A simple structural system (the reinforced concrete girder bridge with a single span) enables to show clearly application and critical comparison of load bearing capacities obtained by applied methods.

The assessment is based on verification of bending moments only due to the lack of information concerning shear reinforcement. However, the benefit of using probabilistic approach is foreseen to be similar as in the case of bending moments.

Methods for assessment of existing bridges

At present existing bridges are mostly assessed by the partial factor method for structural design that can hardly reflect bridge-specific conditions in reliability analysis. Assessments of existing bridges are then often conservative and lead to expensive costs for reconstruction. The assessment of existing road bridge in the Czech Republic is based on determining load bearing capacity $V_i$ (the greatest acceptable weight of each vehicle in the most unfavourable longitudinal position on the bridge) in accordance with ČSN 73 6222 [2].

ČSN 73 6222 [2] assumes three different conditions of crossing for the assessment of load bearing capacities $V_i$:
- $V_1$ is determined for the crossing of a defined two-axle vehicle with a uniform loading representing normal traffic,
- $V_2$ is determined for the crossing of a single three-axle or six-axle vehicle with restricted access of other vehicles. Vehicle with more unfavourable effect is taken into account and
- $V_3$ is determined for the crossing of a nine-axle vehicle with controlled position on a bridge and prescribed speed.

The most unfavourable transversal position of the vehicles for $V_1$ and $V_2$ and of the uniform load for $V_1$ is taken into account.

**Partial factor method.** Application of partial factors for structural design is great disadvantage of this method. Conservative values of these factors have been intentionally proposed to cover most situations in design when information about real material properties or structural geometry is unavailable. Therefore, it may be inappropriate for the assessment of specific existing bridge. According to the partial factor method, load bearing capacity $V_i$ is estimated as follows:

\[
V_i = k_i M_{Qi} \min \left[ \left( \frac{M_{Rd} - \gamma_{G,\text{sup}} M_{Gk}}{\delta_i \psi_{Q1}} \right) \left( \frac{M_{Rd} - \xi \gamma_{G,\text{sup}} M_{Gk}}{\delta_i \gamma_{Q1}} \right) \right] \quad (1)
\]

where

- $k_i$ – is coefficient dependent on type of load bearing capacity $V_i$ derived from ČSN 73 6222 [2],
- $M_{Qi}$ – bending moment from a unity vehicle and uniform loading defined for the different conditions of crossing ($V_1$ to $V_3$) according to ČSN 73 6222 [2],
- $M_{Rd}$ – design value of flexural resistance in accordance with EN 1992-2 [3], using partial factor for $\gamma_C = 1.5$ for concrete and $\gamma_S = 1.15$ for reinforcing steel,
- $\gamma_{G,\text{sup}} = 1.35$ – partial factor for permanent actions,
- $M_{Gk}$ – characteristic bending moment due to permanent actions (self-weight of load bearing structure and road pavement),
- $\delta_i$ – dynamic factor in accordance with ČSN 73 6222 [2],
- $\psi_{Q1} = 0.75$ – combination factor for traffic load,
- $\gamma_{Q1} = 1.35$ – partial factor for traffic load and
- $\xi = 0.85$ – reduction factor.

Partial factors can be updated when information about the bridge is available. This updating is illustrated in a separate contribution [9] and other scientific publications [10-13]. In this study partial factors are not updated due to illustrating of lacks of design procedure which are generally used in the practice.

**Probabilistic method.** Probabilistic assessment is based on reliability index $\beta$ derived from probability of failure $P_f$ in accordance with EN 1990 [4] and ISO 13 822 [5]. The probability of failure (assessed e.g. by the Monte Carlo method) is determined considering the following limit state function:

\[
P_f = P[\theta_R M_R - \theta_E (k_i M_{Qi} + M_G) < 0] \quad (2)
\]

where

- $P[]$ – is probability,
- $\theta_R$ – model uncertainty for resistance and
- $\theta_E$ – model uncertainty for action effects.

Partial factors, characteristic and design values, reduction factors and combination factors are not considered in the probabilistic analysis. Basic variables for the assessment of load bearing capacities $V_i$ are determined by the type of distribution and their statistical characteristics as a mean and coefficient of variation. In case of concrete cover skewness is additionally estimated. The reliability index $\beta$ as an additional measure of reliability is related to the probability of failure through the inverse cumulative distribution function of the standardized normal variable [4].

Load bearing capacity $V_i$ is assessed from Eq. 2 iteratively in order to achieve probability of failure $P_f$ corresponding to the target reliability index $\beta$. Minimal values of $\beta$ and corresponding
target probability of failure $P_t$ in accordance with EN 1990 [4] are provided in Table 1. ISO 13 822 [5] provides a similar, somewhat more detailed reliability differentiation. Target reliability levels for existing buildings and bridges are discussed in details in [13].

Table 1: Minimal values of $\beta$ and $P_t$ for different reliability levels and reference period

<table>
<thead>
<tr>
<th>Reliability level</th>
<th>Reference period 1 year $\beta$</th>
<th>$P_t$</th>
<th>Reference period 50 years $\beta$</th>
<th>$P_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC1</td>
<td>4.2</td>
<td>$10^{-5}$</td>
<td>3.3</td>
<td>$4 \times 10^{-4}$</td>
</tr>
<tr>
<td>RC2</td>
<td>4.7</td>
<td>$10^{-6}$</td>
<td>3.8</td>
<td>$7 \times 10^{-5}$</td>
</tr>
<tr>
<td>RC3</td>
<td>5.2</td>
<td>$10^{-7}$</td>
<td>4.3</td>
<td>$9 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Information about the bridge

Load bearing structure. The single span bridge consists of four main longitudinal reinforced concrete girders stiffened by several transversal beams, reinforced concrete slab and stone masonry abutment. Scheme of the structural system is shown on Fig. 1. Girders are reinforced in the mid-span by 12 steel rods of a diameter 36 mm in three layers with 36mm vertical and horizontal gaps.

![Scheme of the structural system](image)

Fig. 1: Schematic longitudinal section and cross section in the mid-span of the bridge (dimension in mm)

Inspection outcomes. Inspection of the bridge revealed:
- Concrete degradation at the bottom part of both outer longitudinal girders caused by deicing salts and chloride ingress,
- Insignificant corrosion of longitudinal and shear reinforcement and
- Damage of road pavement at about 20 % of the total area, mainly in the area of bridge expansion joints.

No visible degradation and damage was observed at remaining parts of the bridge.

Load effects and structural model. In addition to the traffic loads described above, the bridge is exposed to permanent actions including layers of the road pavement and self-weight of the structural model. According to ČSN 73 6222 [2] thermal and wind effects are neglected.

Load effects (internal forces) are estimated using a slab-wall model developed in Scia Engineer 2012, considering the following simplifications:
- The slab is not inclined,
- The transversal beams are replaced by increasing slab depth by 1 cm,
- Reinforcement of concrete and effect of cornices are neglected,
- Influence of cracks on stiffness is not considered.

Results of tests and measurements. 18 measurements of yield strength of reinforcement $f_y$ include three destructive tests and 15 non-destructive tests by hardness tester. Eight measurements of concrete compressive strength $f_c$ include two destructive tests and six non-destructive tests by Schmidt hammer. Concrete cover $c$ was measured at 59 locations. Statistical characteristics of $f_y, f_c$, and $c$ obtained from the measurements are provided in Table 2.
Assessment of load bearing capacities \( V_i \)

**Basic variables.** Basic variables and their characteristics for the verification by the partial factor method and probabilistic method are given in Table 2. Parameters of basic variables are chosen in accordance with [6-8]. Assessment of load bearing capacities \( V_i \) is evaluated for main load bearing structure (reinforced beams and slab), since road pavement and stone abutment have not significant influence on the reliability of a structure.

Table 2: Basic variables and partial factors

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Partial factor method ( x_{\text{nom}} )</th>
<th>Probabilistic method</th>
<th>Statistical characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
<td>Mean ( \mu_X )</td>
</tr>
<tr>
<td>( a )</td>
<td>Axial distance of beams</td>
<td>1.35 m</td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>Width of the beam</td>
<td>350 mm</td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>Concrete cover</td>
<td>47 mm</td>
<td>BETA ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td>Depth of the slab</td>
<td>150 mm</td>
<td>N ( x_{\text{nom}} )</td>
<td>0.067</td>
</tr>
<tr>
<td>( f_c )</td>
<td>Concrete compressive strength</td>
<td>21.6 MPa</td>
<td>LN0 ( 26.9 \text{ MPa} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( f_y )</td>
<td>Yield strength of reinforcement</td>
<td>257 MPa</td>
<td>LN0 ( 269 \text{ MPa} )</td>
<td>0.025</td>
</tr>
<tr>
<td>( h )</td>
<td>Height of the beam</td>
<td>( x_{\text{nom}} )</td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( A_s )</td>
<td>Longitudinal reinforcement</td>
<td>12214 mm(^2)</td>
<td>N ( 1.02 x_{\text{nom}} )</td>
<td>0.02</td>
</tr>
<tr>
<td>( L )</td>
<td>Length of the beam</td>
<td>23.5 m</td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Reduction factor</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_C )</td>
<td>Partial factor for concrete</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_G )</td>
<td>Partial factor for permanent actions</td>
<td>1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_Q )</td>
<td>Partial factor for traffic load</td>
<td>1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_S )</td>
<td>Partial factor for reinforcement</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>Dynamic factor</td>
<td>( x_{\text{nom}} )</td>
<td>DET ( x_{\text{nom}} )</td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>Reduction factor for permanent action</td>
<td>0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_{RC} )</td>
<td>Reinforced concrete density</td>
<td>2500 kg/m(^3)</td>
<td>N ( x_{\text{nom}} )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \psi_{01} )</td>
<td>Combination factor</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_E )</td>
<td>Model uncertainty for action effects</td>
<td>-</td>
<td>LN0 ( 1 )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \theta_R )</td>
<td>Model uncertainty for resistance</td>
<td>-</td>
<td>LN0 ( 1.075 )</td>
<td>0.075</td>
</tr>
</tbody>
</table>

1 Height of the beam is depended on the distance of cross section from the support \( x \); \( h(x=0) = 1 \text{ m} \); \( h(x=11.75) = 1.3 \text{ m} \)

2 Dynamic factor is dependent on the type of crossing \( V_i \) and the first natural frequency in accordance with ČSN 73 6222 [2]; in this study \( \delta(V_1) = 1.35, \delta(V_2) = 1.35 \) and \( \delta(V_3) = 1.05 \)

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BETA distribution is used for concrete cover. The four parameter distribution is defined by a mean value, coefficient of variation, skewness and origin. A value of skewness \( \omega_c = 0.156 \) as obtained from measurements and the origin at zero are taken into account. For the partial factor method the characteristic values of yield strength of reinforcement and concrete compressive strength are determined as a 5% lower fractile of a two parameter lognormal distribution. The nominal values of the bending moment \( M_Q \) due to the unity vehicle(s) and uniform loading if applicable are \( M_Q(V_1) = 304 \text{ kNm}, M_Q(V_2) = 1.28 \text{ kNm} \) and \( M_Q(V_3) = 1.04 \text{ kNm} \). In the probabilistic assessment these values are considered deterministic since the variable \( \theta_E \) is assumed to involve uncertainties in load effects due to crossing of the vehicles.

**Assessment by the partial factor method.** Load bearing capacities \( V_i \) are estimated by the partial factor method for structural design for all cross sections of each longitudinal girder. Due to the symmetry of the bridge load bearing capacities \( V_i \) are same for the pairs of the inner and outer girders. The inner girders have smaller load bearing capacities \( V_i \) and consequently load bearing capacities of the inner girders are discussed hereafter only.
Self-weight of the load bearing structure is estimated on the basis of cross-section characteristics and concrete volume density of 24 kNm$^{-3}$. Other permanent actions are described by a uniform loading with the characteristic value of 0.65 kNm$^{-2}$. Vehicles are defined by crossing of axle loads with respect to the considered crossing conditions described in above.

Fig. 2 shows the variability of load bearing capacity $V_1$ given in tons with the distance from support for the partial factors for structural design. In addition the figure illustrates identification of the critical cross section where $V_1$ is minimised. Similar trends are observed for the load bearing capacities $V_2$ and $V_3$. Table 3 gives load bearing capacities $V_i$ assessed by the partial factor method and the distances of critical cross sections from the support.

![Fig. 2: Variability of load bearing capacity $V_1$ with the distance from supports for the partial factors for structural design](image)

<table>
<thead>
<tr>
<th>Critical cross section [m]</th>
<th>Load bearing capacity [t]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>10</td>
</tr>
<tr>
<td>$V_2$</td>
<td>10</td>
</tr>
<tr>
<td>$V_3$</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Load bearing capacities $V_i$ are different in each cross section (see Fig. 2), since load bearing capacity of the bridge is the smallest value in critical cross section. Critical cross section is not in the mid-span due to crossing of axle loads and geometry of the girders. Load bearing capacity $V_1$ is the smallest while $V_3$ attains the highest values.

**Assessment by probabilistic method.** Assessment of load bearing capacity $V_i$ for probabilistic method in whole length of the bridge is difficult due to considerable computational demands. Therefore, the probabilistic assessment is conducted only for the critical cross sections identified by the partial factor method.

The statistical characteristics of basic variables applied in the probabilistic analysis are given in Table 2. ISO 13 822 [5] provides a more complex range of the target reliability index $\beta$ than EN 1990 [4] as the target levels are indicated for four consequence classes. Load bearing capacities $V_i$ for different $\beta$ assessed by the probabilistic method are summarized in Table 4.

![Table 3: Load bearing capacities $V_i$ in accordance with the partial factor method](image)

<table>
<thead>
<tr>
<th>Critical cross section [m]</th>
<th>Load bearing capacity [t]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>66</td>
</tr>
<tr>
<td>$V_2$</td>
<td>100</td>
</tr>
<tr>
<td>$V_3$</td>
<td>188</td>
</tr>
</tbody>
</table>

![Table 4: Load bearing capacities $V_i$ obtained by the probabilistic method](image)
It appears that load bearing capacities $V_i$ decrease with an increasing reliability index $\beta$. When statistical parameters of basic variables can be determined with sufficient accuracy, then the probabilistic method improves estimates of load bearing capacities as compared to the partial factor method for structural design.

**Comparison of load bearing capacities**

Fig. 3 shows comparison of load bearing capacities $V_i$ for the partial factor method and probabilistic method. It appears that the partial factor method provides conservative results. Probabilistic method leads to about 40% higher results of load bearing capacities $V_i$ for the common target reliability index, $\beta = 3.8$ (medium failure consequences according to ISO 13822 [5]) than the partial factor method. Fig. 3 indicates that the target reliability index is inadequately reflected by the assessment by partial factors for structural design. For the probabilistic method the reliability index $\beta$ significantly influences the load bearing capacities $V_i$ that decrease with increasing $\beta$.

The partial factor method for structural design is intended to be conservative in most cases of practical relevance while the probabilistic method takes into account actual conditions of a bridge and should enhance estimates of a load bearing capacity. That is why the load bearing capacities $V_i$ assessed by the probabilistic method should exceed those obtained by the partial factor method. However, the probabilistic method could provide lower values of $V_i$ in cases of severe actual conditions of the bridge beyond the scope of considerations behind common design methods.

Note that the presented study is focused on the Ultimate limit state related to flexural failure mode. With respect to another common failure mode due to shear forces, it is foreseen that the benefit of using the probabilistic method would be similar as in the case of bending moments.

**Conclusions**

The load bearing capacity of an existing reinforced concrete bridge from 1908 is assessed in this study in accordance with valid standards using the partial factor method and probabilistic approach. Critical comparison of obtained load bearing capacities reveals that:

- When compared with the probabilistic approach, the partial factor method:
  - Is more operational as it is easier to compute and requires less experience of designer with the theory of structural reliability,
  - Provides less accurate and mostly conservative results when using the partial factors for structural design.
Probabilistic method enables to adequately account for uncertainties related to assessment of existing structures and reflect a specified target reliability.
- Load bearing capacities $V_i$ assessed by probabilistic method are about 40% higher for the most common target reliability index $\beta = 3.8$.

Acknowledgement

The study is a part of the research project NAKI DF12P01OVV040 supported by the Ministry of Culture of the Czech Republic. The study is based on the diploma thesis by Jan Krejsa.

References